Model of Strongly Correlated 2D Fermi Liquids Based on Fermion-Condensation Quantum Phase Transition

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Abstract

A model of strongly correlated electron or hole liquids with the fermion condensate is presented and applied to the consideration of quasiparticle excitations in high temperature superconductors, in their superconducting and normal states. Within our model the appearance of the fermion condensate presents a quantum phase transition, that separates the regions of normal and strongly correlated electron liquids. Beyond the fermion condensation quantum phase transition point the quasiparticle system is divided into two subsystems, one containing normal quasiparticles and the other — fermion condensate localized at the Fermi surface. In the superconducting state the quasiparticle dispersion in systems with FC can be presented by two straight lines, characterized by effective masses M^*_{FC} and M^*_L, respectively, and intersecting near the binding energy which is of the order of the superconducting gap. This same quasiparticle picture persists in the normal state, thus manifesting itself over a wide range of temperatures as new energy scales. Arguments are presented that fermion systems with FC have features of a "quantum protectorate". A theory of high temperature superconductivity based on the combination of the fermion-condensation quantum phase transition and the conventional theory of superconductivity is presented. This theory describes maximum values of the superconducting gap which can be as big as $\Delta_1 \sim 0.1\varepsilon_F$, with ε_F being the Fermi level. We show that the critical temperature $2T_c \simeq \Delta_1$. If there exists the pseudogap above T_c then $2T^* \simeq \Delta_1$, and T^* is the temperature at which the pseudogap vanishes. A discontinuity in the specific heat at T_c is calculated. The transition from conventional superconductors to high-T_c ones as a function of the doping level is investigated. The single-particle excitations and their lineshape are also considered. Analyzing experimental data on the high temperature superconductivity in different materials induced by field-effect doping, we show that all these facts can be understood within a theory of the superconductivity based on the fermion condensation quantum phase transition, which can be conceived of as a universal cause of the superconductivity. The main features of a room-temperature superconductor are considered.

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I. INTRODUCTION

One of the most challenging problems of modern physics is the problem of systems with a big coupling constant. This problem is of crucial importance particularly in the quantum field theory, making even the quantum electrodynamics to be not a self-consistent theory [1]. It is well-known that a consideration of strongly correlated liquids is close to the problem of systems with the big coupling constant which persists in many-body physics. A solution of this problem has been offered in the context of the Landau theory of normal Fermi liquids by introducing into the theory a notion of the quasiparticles and parameters which characterize the effective interaction between the quasiparticles [2]. As a result, the Landau theory has removed high energy degrees of freedom and kept a sufficiently large number of relevant low energy degrees of freedom to treat liquid's low energy properties. Usually, it is assumed that the breakdown of the Landau theory is defined by the Pomeranchuk stability conditions and occurs when the Landau amplitudes being negative reach its critical value. Note that the new phase at which the stability conditions are restored can in principle be again described in the framework of the theory. To describe a strongly correlated electron liquid, taking place when the coupling constant becomes sufficiently big, a conventional way can be used, assuming that the correlated regime is connected to the noninteracting Fermi gas by adiabatic continuity in the same way as in the framework of the Landau normal Fermi liquid theory [2]. But a question exists whether this is possible at all. Most likely, the answer is negative. Therefore, we direct our attention to a model, in the frame of which a strongly correlated electron liquid is separated from conventional Fermi liquid by a phase transition related to the onset of the fermion condensate (FC) [3,4]. It was demonstrated rather recently [3] that the Pomeranchuk conditions are covering not all possible limitations: one is overlooked, being connected with the situation when, at temperature $T = 0$, the effective mass can become infinitely big. It has been demonstrated that such a situation, leading to profound consequences, can take place when the corresponding amplitude being positive reaches the critical value, producing a completely new class of strongly correlated Fermi liquids with FC [3,4] which is separated from a normal Fermi liquid by the fermion condensation quantum phase transition (FCQPT) [5]. In this case we are dealing with the strong coupling limit where an absolutely reliable answer cannot be given, based on pure theoretical first principle ground. Therefore, the only way to verify that FC occurs is to consider experimental facts which bear witness to the existence of such a state. We assume that these facts can be find in two-dimensional (2D) systems with interacting electrons or holes, which can be presented by modulation doped quantum wells, by high mobility metal-oxide-semiconductor field effect transistors, or by high-T_c superconductors.

The aim of our report is to show that within the frameworks of our model of a strongly correlated electron (hole) liquid based on FCQPT the main properties of such liquids observed in the high-temperature superconductors can be understood. In Sec. II, we review the general features of Fermi liquids with FC, showing that an electron liquid of low density in the high-T_c materials inevitably undergoes FCQPT. In Sec. III we consider the high-temperature superconductivity, which takes place in the presence of FC. In Sec. IV we describe the quasiparticle dispersion and lineshape. Finally, in Sec. V, we summarize our main results.

II. THE GENERAL FEATURES OF FERMI LIQUIDS WITH FC

Let us start by considering the key points of the FC theory. FC is a new solution of the Fermi liquid theory equations [2] for the quasiparticle occupation numbers $n(p, T)$

$$\frac{\delta(F-\mu N)}{\delta n(p,T)} = \varepsilon(p,T) - \mu(T) - T\ln\frac{1-n(p,T)}{n(p,T)} = 0, \tag{1}$$

which depends on the momentum p and temperature T. Here F is the free energy, and μ is the chemical potential, while

$$\varepsilon(p,T) = \frac{\delta E[n(p)]}{\delta n(p,T)}, \tag{2}$$

is the quasiparticle energy. This energy is a functional of $n(p,T)$ just like the total energy $E[n(p)]$ and the other thermodynamic functions. Eq. (1) is usually presented as the Fermi-Dirac distribution,

$$n(p,T) = \left\{1 + \exp\left[\frac{(\varepsilon(p,T)-\mu)}{T}\right]\right\}^{-1}. \tag{3}$$

At $T \to 0$ one gets from Eqs. (1), (3) the standard solution $n_F(p, T \to 0) \to \theta(p_F - p)$, with $\varepsilon(p \simeq p_F) - \mu = p_F(p-p_F)/M_L^*$, where p_F is the Fermi momentum, and M_L^* is the Landau effective mass [2],

$$\frac{1}{M_L^*} = \frac{1}{p}\frac{d\varepsilon(p,T=0)}{dp}|_{p=p_F}. \tag{4}$$

It is implied that M_L^* is positive and finite at the Fermi momentum p_F. As a result, the T-dependent corrections to M_L^*, to the quasiparticle energy $\varepsilon(p)$, and other quantities, start with T^2-terms. But this solution is not the only one possible. There exist "anomalous" solutions of Eq. (1) associated with the so-called fermion condensation [3,6]. Being continuous and satisfying the inequality $0 < n(p) < 1$ within some region in p, such solutions $n(p)$ admit a finite limit for the logarithm in Eq. (1) at $T \to 0$ yielding,

$$\varepsilon(p) = \frac{\delta E[n(p)]}{\delta n(p)} = \mu;\ p_i \le p \le p_f. \tag{5}$$

At $T = 0$ Eq. (5) determines FCQPT, possessing solutions at some $r_s = r_{FC}$ as soon as the effective inter-electron interaction becomes sufficiently strong [7]. From here on we shall call a hole or electron system as electron one provided this will not lead to confusion. In a simple electron liquid, the effective inter-electron interaction is proportional to the dimensionless average interparticle distance $r_s = r_0/a_B$, with $r_0 = \sqrt{2}/p_F$ being the average distance, and a_B is the Bohr radius. Equation (5) leads to the minimal value of E as a functional of $n(p)$ when in system under consideration a strong rearrangement of the single particle spectra can take place. We see from Eq. (5) that the occupation numbers $n(p)$ become variational parameters: the solution $n(p)$ takes place if the energy E can be lowered by alteration of the occupation numbers. Thus, within the region $p_i < p < p_f$, the solution $n(p) = n_F(p) + \delta n(p)$ deviates from the Fermi step function $n_F(p)$ in such a way that the energy $\varepsilon(p)$ stays constant while outside this region $n(p)$ coincides with $n_F(p)$. It is pertinent to note that the above general consideration was verified by inspecting simple models. As the result, it was shown that the onset of the FC does lead to lowering the free energy [6,8]. It follows from the above consideration that the superconductivity order parameter $\kappa(\mathbf{p}) = \sqrt{n(\mathbf{p})(1-n(\mathbf{p}))}$ has a nonzero value over the region occupied by FC. The superconducting gap $\Delta(\mathbf{p})$ being linear in the coupling constant of the particle-particle interaction V_{pp} gives rise to the high value of T_c because

one has $2T_c \simeq \Delta$ [8] within the standard Bardeen-Cooper-Schrieffer (BCS) theory [9]. As it is shown in Sec. III, if the superconducting gap $\Delta \neq 0$, the quasiparticle effective mass becomes finite. In consequence of these features the density of states at the Fermi level becomes finite and the involved quasiparticles are not localized. On the other hand, even at $T = 0$, Δ can vanish, provided V_{pp} is repulsive or absent. Then, as it is seen from Eq. (5), the Landau quasiparticle system becomes separated into two subsystems. The first contains the Landau quasiparticles, while the second, related to FC, is localized at the Fermi surface and formed by dispersionless quasiparticles. As a result, the standard Kohn-Sham scheme for the single particle equations is no longer valid beyond the point of the FC phase transition [10]. Such a behavior of systems with FC is clearly different from what one expects from the well known local density calculations. Therefore these calculations are hardly applicable to describe systems with FC. It is also seen from Eq. (5) that a system with FC has a well-defined Fermi surface.

Let us assume that FC has just taken place, that is $p_i \to p_f \to p_F$, and the deviation $\delta n(p)$ is small. Expanding functional $E[n(p)]$ in Taylor's series with respect to $\delta n(p)$ and retaining the leading terms, one obtains from Eq. (5),

$$\mu = \varepsilon(\mathbf{p}) = \varepsilon_0(\mathbf{p}) + \int F_L(\mathbf{p}, \mathbf{p}_1)\delta n(\mathbf{p_1})\frac{d\mathbf{p}_1}{(2\pi)^2};\ p_i \leq p \leq p_f, \tag{6}$$

where $F_L(\mathbf{p}, \mathbf{p}_1) = \delta^2 E/\delta n(\mathbf{p})\delta n(\mathbf{p}_1)$ is the Landau interaction. Both the Landau interaction and the single-particle energy $\varepsilon_0(p)$ are calculated at $n(p) = n_F(p)$. It is seen from Eq. (6) that the FC quasiparticles forms a collective state, since their energies are defined by the macroscopical number of quasiparticles within the region $p_i - p_f$, and vice versa. The shape of the spectra is not effected by the Landau interaction, which, generally speaking, depends on the system's properties, including the collective states, impurities, etc. The only thing defined by the interaction is the width of the region $p_i - p_f$, provided the interaction is sufficiently strong to produce the FC phase transition at all. Thus, we can conclude that the spectra related to FC are of universal form, being dependent, as we will see below, mainly on temperature T, if $T > T_c$, or on the superconducting gap at $T < T_c$.

According to Eq. (1), the single-particle excitations within the interval $p_i - p_f$ have at $T_c \leq T \ll T_f$ the shape $\varepsilon(p, T)$ linear in T [8,11], which can be simplified at the Fermi level,

$$\varepsilon(p, T) - \mu(T) = T \ln \frac{1 - n(p)}{n(p)} \simeq T\frac{1 - 2n(p)}{n(p)}|_{p \simeq p_F}. \tag{7}$$

T_f is the temperature, above which FC effects become insignificant [8],

$$\frac{T_f}{\varepsilon_F} \sim \frac{p_f^2 - p_i^2}{2M\varepsilon_F} \sim \frac{\Omega_{FC}}{\Omega_F}. \tag{8}$$

Here Ω_{FC} is the FC volume, ε_F is the Fermi energy, and Ω_F is the volume of the Fermi sphere. We note that at $T_c \leq T \ll T_f$ the occupation numbers $n(p)$ are approximately independent of T, being given by Eq. (5). One can imagine that at these temperatures dispersionless plateau $\varepsilon(p) = \mu$ given by Eq. (5) is slightly turned counter-clockwise about μ. As a result, the plateau is just a little tilted and rounded off at the end points. According to Eq. (7) the effective mass M^*_{FC} related to FC is given by,

$$M^*_{FC} \simeq p_F\frac{p_f - p_i}{4T}. \tag{9}$$

To obtain Eq. (9) an approximation for the derivative $dn(p)/dp \simeq -1/(p_f - p_i)$ was used. Having in mind that $p_f - p_i \ll p_F$, and using (8) and (9) the following estimates for the effective mass M^*_{FC} are obtained,

$$\frac{M^*_{FC}}{M_0} \sim \frac{N(0)}{N_0(0)} \sim \frac{T_f}{T}. \tag{10}$$

Eqs. (9) and (10) show the temperature dependence of M^*_{FC}. In (10) M_0 denotes the bare electron mass, $N_0(0)$ is the density of states of noninteracting electron gas, and $N(0)$ is the density of states at the Fermi level. Multiplying both sides of Eq. (9) by $p_f - p_i$ we obtain the energy scale E_0 separating the slow dispersing low energy part, related to the effective mass M^*_{FC}, from the faster dispersing relatively high energy part, defined by the effective mass M^*_L [5,12],

$$E_0 \simeq 4T. \tag{11}$$

It is seen from Eq. (11) that the scale E_0 does not depend on the condensate volume. The single particle excitations are defined according to Eqs. (7) and (9) by the temperature and by $n(p)$, given by Eq. (5). Thus, we are led to the conclusion that the one-electron spectrum is negligible disturbed by thermal excitations, impurities, etc, so that one observes the features of the quantum protectorate [13,14].

It is seen from Eq. (5) that at the point of FC phase transition $p_f \to p_i \to p_F$, M^*_{FC} and the density of states, as it follows from Eqs. (5), (10), tend to infinity. One can conclude that at $T = 0$ and as soon as $r_s \to r_{FC}$, FCQPT takes place being connected to the absolute growth of M^*_L. It is essential to have in mind, that the onset of the charge density wave instability in a many-electron system, such as electron liquid, which takes place as soon as the effective inter electron constant reaches its critical value $r_s = r_{cdw}$, is preceded by the unlimited growth of the effective mass. Therefore, the FC occurs before the onset of the charge density wave. Hence, at $T = 0$, when r_s reaches its critical value $r_{FC} < r_{cdw}$, the FCQPT inevitably takes place [7]. It is pertinent to note that this growth of the effective mass with decreasing electron density was observed experimentally in a metallic 2D electron system in silicon at $r_s \simeq 7.5$ [15]. Therefore we can take $r_{FC} \sim 7.5$. On the other hand, there exist charge density waves or strong fluctuations of charge ordering in underdoped high-T_c superconductors [16]. Thus, the formation of FC in high-T_c compounds can be thought as a general property of an electron liquid of low density which is embedded in these solids, rather then an uncommon and anomalous solution of Eq. (1) [7]. Beyond the point of FCQPT the condensate volume is proportional to $(r_s - r_{FC})$ as well as $T_f/\varepsilon_F \sim (r_s - r_{FC})/r_{FC}$ at least when $(r_s - r_{FC})/r_{FC} \ll 1$. Note, that such a behavior is in accordance with the general properties of second order phase transitions. Therefore, we can accept a model relating systems with FC to high-T_c compounds, assuming that the effective coupling constant r_s increases with decreasing doping, exceeding its critical value r_{FC} at the levels corresponding to optimal doped samples. We remark, that this critical value r_{FC} corresponds to the r_s values of highly overdoped samples [7]. As the result, our quite natural model suggests that both quantities, T_f and condensate volume Ω_{FC}, increase with decrease of doping. Thus, these values are higher in underdoped samples as compared to overdoped ones provided r_s meets the mentioned above conditions. While, in the highly overdoped regime only slight deviations from the normal Fermi liquid are observed [17]. All these peculiar properties are naturally explained within a model proposed in [5,12,18,19] and allow to relate the doping level x regarded as the density of mobile charge carriers (holes or electrons) per unit area to the density of Fermi liquid with FC. We assume that x_{FC} corresponds to the highly overdoped regime at which

FCQPT takes place, and introduce the effective coupling constant $g_{eff} \sim (x - x_{FC})/x_{FC}$. In our model, the doping level x at $x \leq x_{FC}$ in metals is related to $(p_f - p_i)$ in the following way:

$$g_{eff} \sim \frac{(x_{FC} - x)}{x_{FC}} \sim \frac{(p_f - p_i)(p_f + p_i)}{p_F^2} \sim \frac{p_f - p_i}{p_F}. \tag{12}$$

According to experimental facts the large density of states at the Fermi level reaches its maximum in the vicinity of the Hove singularities, that is around the point $(\pi, 0)$ of the Brillouin zone, or $\bar{M}$, in high-T_c compounds. The density of states reaches its minimum value at the intersection of the so called nodal direction of the Brillouin zone with the Fermi surface (see, e.g., [20]). The FC sets in around the van Hove singularities [11], causing, according to Eqs. (9) and (10), large density of states and large value of the difference $(p_f - p_i)$ at the point $\bar{M}$. Then, the volume Ω_{FC} and difference $(p_f - p_i)$ start to depend on the point of the Fermi surface, say, on the angle ϕ along the Fermi surface, which we count from the point $\bar{M}$ to the nodal direction. Nonetheless, as it is seen from Eq. (11), E_0 remains constant, being independent of the angle. It is not the case for the effective mass M^*_{FC}, that can strongly depend upon the angle via the difference $(p_f(\phi) - p_i(\phi))$ increasing from the nodal direction towards $\bar{M}$, as it follows from Eq. (9). It is pertinent to note that outside the FC region the single particle spectrum is negligible affected by the temperature, being defined by M^*_L. Thus, we come to the conclusion that a system with FC is characterized by two effective masses: M^*_{FC} that is related to the single particle spectrum at lower energy scale, and M^*_L describing the spectrum at higher energy scale. These two effective masses manifest itself as a break in the quasiparticle dispersion, which can be approximated by two straight lines intersecting at the energy E_0. This break takes place at temperatures $T_c \leq T \ll T_f$ in accordance with the experimental findings [21], and, as we will see, at $T \leq T_c$ corresponding to the experimental facts [21,22]. As to the quasiparticle formalism, it is applicable to this problem since the width γ of single particle excitations is not large compared to their energy being proportional $\gamma \sim T$ at $T > T_c$ [8]. The lineshape can be approximated by a simple Lorentzian [12], being in accordance with experimental data obtained from scans at a constant binding energy [23], see Sec. IV. Then, FC serves as a stimulating source of new phase transitions which lift the degeneration of the spectrum. For example, FC can generate the spin density wave, or antiferromagnetic phase transition, thus leading to a whole variety of the system's properties. Then, the onset of the charge density wave is preceded by FCQPT, and both of these phases can coexist at the sufficiently low density when $r_s \geq r_{cdw}$. The simple consideration presented above explains extremely large variety of properties of high-T_c superconductors. We have seen above that the superconductivity is strongly aided by FC, because both of the phases are characterized by the same order parameter. As a result, the superconductivity, removing the spectrum degeneration, "wins the competition" with the other phase transitions up to the critical temperature T_c. We turn now to a consideration of both the superconducting state and quasiparticle dispersions at $T \leq T_c$.

III. THE SUPERCONDUCTING STATE

The explanation of the large values of the critical temperature T_c, of the maximum value of the superconducting gap Δ_1, of the relation between Δ_1 and the temperature T^* at which the pseudogap vanishes are, as years before, among the main problems in the physics of high-temperature superconductivity. To solve them, one needs to know the single-particle spectra of corresponding metals. Recent studies of photoemission spectra in copper oxide based compounds discovered an energy scale in the spectrum of low-energy electrons in copper oxides, which manifests itself as a kink

in the single-particle spectra [21–24]. As a result, the spectra in the energy range (-200—0) meV can be described by two straight lines intersecting at the binding energy $E_0 \sim (50 - 70)$ meV [21,22]. The existence of the energy scale E_0 could be attributed to the interaction between electrons and the collective excitations, for instance, phonons [24]. On the other hand, the analysis of the experimental data on the single-particle electron spectra demonstrates that the perturbation of the spectra by phonons or other collective states is in fact very small, therefore, the corresponding state of electrons has to be described as a strongly collectivized quantum state and was named "quantum protectorate" [13,14]. Thus, the interpretation of the above mentioned kink as a consequence of electron-phonon interaction can very likely be in contradiction with the quantum protectorate concept. To describe the large values of T_c, the single-particle spectra and the kink, the assumption can be used that the electron system of high-T_c superconductor has undergone FCQPT.

The compounds are extremely complex materials having a great number of competing degrees of freedom which produce a great variety of physical properties. In turn, these properties can compete and coexist with the superconductivity hindering the understanding of the universal cause of the superconductivity. As a result, it was suggested that the unique superconducting properties in these compounds are defined by the presence of the Cu-O planes, by the d-wave pairing symmetry, and by the existence of the pseudogap phenomena in optimally doped and underdoped cuprates, see e.g. [25–27]. However, recent studies of quasiparticle tunneling spectra of cuprates have revealed that the pairing symmetry may change from the d to s-wave symmetry, depending on the hole, or electron, doping level [28–30]. Then, the high temperature s-wave superconductivity has been observed in electron doped infinite layer cuprates [31] with a sharp transition at $T = 43$ K and the absence of pseudogap [32]. Therefore, we can conclude that the d-wave symmetry and the pseudogap phenomena are not integral parts of . After all, recent studies of high-T_c superconductivity in C_{60} crystals [33] with the use of field-induced doping, when an increase in the superconducting transition temperature T_c to 52 K was achieved in hole doped samples, have shown that the presence of the Cu-O planes is not the necessary condition to observe high-T_c superconductivity. Then, in lattice expanded C_{60} by intercalating $CHCl_3$ and $CHBr_3$ into the lattice, the higher T_c of 80 K in hole doped $C_{60}/CHCl_3$ and of 117 K in $C_{60}/CHBr_3$ were observed [34]. In the electron-doped case, $T_c = 11$ K is reached for C_{60} crystals, $T_c = 18$ K and $T_c = 26$ K were observed in samples intercalated with $CHCl_3$ and $CHBr_3$ respectively [34,35]. The described above technique, when the corresponding dopant densities x of electrons or holes are induced by gate doping in a field-effect transistor geometry, permits constructing the variation in $T_c^{\alpha\gamma}(x)$ as a function of x in a wide region of the doping variation [33–35]. Here α denotes the material, say C_{60} or intercalated C_{60}, etc., and γ denotes hole or electron doping. One important point to remember is that in case of electron doped metals we have to treat x as the density of the mobile charge carriers, which corresponds to a narrow variation region of x around half-filling of the conduction band. While the limitation of the hole doping variation is defined by the electric breakdown strength of the gate oxide taking place at a sufficiently high level of the doping [34]. This technique allows the study of properties of metals in question as a function of the doping level x without inducing disorder or possible defects which could have a strong impact on the superconductivity. It is very essential to note that the shape of the functions $T_c^{\alpha\gamma}(x)$ is similar in samples with or without intercalation, that is the shape does not depend on both α and γ [34]. Moreover, this observation is also valid in the case of the field induced superconductivity in both a spin-ladder cuprate $[CaCu_2O_3]_4$ [36] and $CaCuO_2$ [37]. Thus, we can use a simple approximation

$$T_c^{\alpha\gamma}(x) = T_1^{\alpha} T_2^{\gamma}(x_1 - x)x, \tag{13}$$

where the coefficients T_1^{α} and T_2^{γ} define the transition temperature T_c for a given hole (or electron)

metal, and x is the density of the mobile charge carriers with x is obviously tending continuously to zero at the insulator-metal transition. It is directly follows from Eq. (13) that the transition temperature reaches its maximum value T_c^M at the optimal doping level x_{opt}

$$T_c^M = T_c^{\alpha\gamma}(x_{opt}) = T_1^{\alpha} T_2^{\gamma} \left(\frac{x_1}{2}\right)^2. \tag{14}$$

Now we can calculate the value of x_{opt} for the different hole and electron metals studied in [33–37] in terms of the dimensionless parameter r_s, $\pi r_s^2 a_B^2 = 1/x$. We have that $r_s^{opt} \sim 10$ and, thus, r_s^{opt} is approximately independent of the metals. As a result we can recognize that these striking experimental facts, the general shape of the function $T_c(x)$ and the constant value of r_s^{opt}, point to a fact, that the generic properties of high-T_c superconductivity are defined by the 2D charge (electron or hole) strongly correlated liquid rather then by solids which hold this liquid. While the solids arrange the presence of the pseudogap phenomena, the s or d-wave pairing symmetry, the electron-phonon coupling constant defining T_c, the variation region of x, and so on.

At $T = 0$, the ground state energy $E_{gs}[\kappa(\mathbf{p}), n(\mathbf{p})]$ of 2D electron liquid is a functional of the order parameter of the superconducting state $\kappa(\mathbf{p})$ and of the quasiparticle occupation numbers $n(\mathbf{p})$ and is determined by the known equation of the weak-coupling theory of superconductivity, see e.g. [38]

$$E_{gs} = E[n(\mathbf{p})] + \int \lambda_0 V(\mathbf{p}_1, \mathbf{p}_2) \kappa(\mathbf{p}_1) \kappa^*(\mathbf{p}_2) \frac{d\mathbf{p}_1 d\mathbf{p}_2}{(2\pi)^4}. \tag{15}$$

Here $E[n(\mathbf{p})]$ is the ground-state energy of normal Fermi liquid, $n(\mathbf{p}) = v^2(\mathbf{p})$ and $\kappa(\mathbf{p}) = v(\mathbf{p})\sqrt{1 - v^2(\mathbf{p})}$. It is assumed that the pairing interaction $\lambda_0 V(\mathbf{p}_1, \mathbf{p}_2)$ is weak. Minimizing E_{gs} with respect to $\kappa(\mathbf{p})$ we obtain the equation connecting the single-particle energy $\varepsilon(\mathbf{p})$ to $\Delta(\mathbf{p})$,

$$\varepsilon(\mathbf{p}) - \mu = \Delta(\mathbf{p}) \frac{1 - 2v^2(\mathbf{p})}{2\kappa(\mathbf{p})}, \tag{16}$$

here the single-particle energy $\varepsilon(\mathbf{p})$ is determined by the Landau equation (2). The equation for the superconducting gap $\Delta(\mathbf{p})$ takes form

$$\Delta(\mathbf{p}) = -\int \lambda_0 V(\mathbf{p}, \mathbf{p}_1) \kappa(\mathbf{p}_1) \frac{d\mathbf{p}_1}{4\pi^2} = -\frac{1}{2} \int \lambda_0 V(\mathbf{p}, \mathbf{p}_1) \frac{\Delta(\mathbf{p}_1)}{\sqrt{(\varepsilon(\mathbf{p}_1) - \mu)^2 + \Delta^2(\mathbf{p}_1)}} \frac{d\mathbf{p}_1}{4\pi^2}. \tag{17}$$

If $\lambda_0 \to 0$, then, the maximum value $\Delta_1 \to 0$, and Eq. (16) reduces to Eq. (5) [3]

$$\varepsilon(\mathbf{p}) - \mu = 0, \text{ if } 0 < n(\mathbf{p}) < 1;\ p_i \le p \le p_f. \tag{18}$$

Now we can study relationships between the state defined by Eq. (18) and the superconductivity. At $T = 0$, Eq. (18) defines a particular state of Fermi liquid with FC for which the modulus of the order parameter $|\kappa(\mathbf{p})|$ has finite values in the L_{FC} range of momenta $p_i \le p \le p_f$, and $\Delta_1 \to 0$ in the L_{FC}. Such a state can be considered as superconducting, with infinitely small value of Δ_1 so that the entropy of this state is equal to zero. It is obvious, that this state, being driven by the quantum phase transition, disappears at $T > 0$ [5]. When $p_i \to p_F \to p_f$, Eq. (18) determines the point r_{FC} at which the FCQPT takes place. It follows from Eq. (18) that the system brakes into two quasiparticle subsystems: the first subsystem in the L_{FC} range is occupied by the quasiparticles

with the effective mass $M^*_{FC} \propto 1/\Delta_1$, while the second one is occupied by quasiparticles with finite mass M^*_L and momenta $p < p_i$. If $\lambda_0 \neq 0$, Δ_1 becomes finite, leading to a finite value of the effective mass M^*_{FC} in L_{FC}, which can be obtained from Eq. (16) [5,12]

$$M^*_{FC} \simeq p_F \frac{p_f - p_i}{2\Delta_1}. \tag{19}$$

As to the energy scale, it is determined by the parameter E_0:

$$E_0 = \varepsilon(\mathbf{p}_f) - \varepsilon(\mathbf{p}_i) \simeq 2\frac{(p_f - p_F)p_F}{M^*_{FC}} \simeq 2\Delta_1. \tag{20}$$

It is reasonably safe to suggest that we have come back to the Landau theory by integrating out high energy degrees of freedom and introducing the quasiparticles. The sole difference between the Landau Fermi liquid and Fermi liquid undergone FCQPT is that we have to expand the number of relevant low energy degrees of freedom by adding both a new type of quasiparticles with the effective mass M^*_{FC}, given by Eq. (19), and the energy scale E_0 given by Eq. (20). We have also to bear in mind that the properties of these new quasiparticles of a Fermi liquid with FC cannot be separated from the properties of the superconducting state, as it follows from Eqs. (16), (19) and (20). We may say that the quasiparticle system in the range L_{FC} becomes very "soft" and is to be considered as a strongly correlated liquid. On the other hand, the system's properties and dynamics are dominated by a strong collective effect having its origin in FCQPT and determined by the macroscopic number of quasiparticles in the range L_{FC}. Such a system cannot be disturbed by the scattering of individual quasiparticles and has features of a quantum protectorate [5,13,14].

We assume that the range L_{FC} is small, $(p_f - p_F)/p_F \ll 1$, and $2\Delta_1 \ll T_f$ so that the order parameter $\kappa(\mathbf{p})$ is governed mainly by the FC [5,18]. To solve Eq. (17) analytically, we take the Bardeen-Cooper-Schrieffer (BCS) approximation for the interaction [9]: $\lambda_0 V(\mathbf{p}, \mathbf{p}_1) = -\lambda_0$ if $|\varepsilon(\mathbf{p}) - \mu| \leq \omega_D$, the interaction is zero outside this region, with ω_D being the characteristic phonon energy. As a result, the gap becomes dependent only on the temperature, $\Delta(\mathbf{p}) = \Delta_1(T)$, being independent of the momentum, and Eq. (17) takes the form

$$1 = N_{FC}\lambda_0 \int_0^{E_0/2} \frac{d\xi}{\sqrt{\xi^2 + \Delta_1^2(0)}} + N_L\lambda_0 \int_{E_0/2}^{\omega_D} \frac{d\xi}{\sqrt{\xi^2 + \Delta_1^2(0)}}. \tag{21}$$

Here we set $\xi = \varepsilon(\mathbf{p}) - \mu$ and introduce the density of states N_{FC} in the L_{FC}, or E_0, range. As it follows from Eq. (19), $N_{FC} = (p_f - p_F)p_F/2\pi\Delta_1(0)$. The density of states N_L in the range $(\omega_D - E_0/2)$ has the standard form $N_L = M^*_L/2\pi$. If the energy scale $E_0 \to 0$, Eq. (21) reduces to the BCS equation. On the other hand, assuming that $E_0 \leq 2\omega_D$ and omitting the second integral in the right hand side of Eq. (21), we obtain

$$\Delta_1(0) = \frac{\lambda_0 p_F(p_f - p_F)}{2\pi} \ln(1 + \sqrt{2}) = 2\beta\varepsilon_F \frac{p_f - p_F}{p_F} \ln(1 + \sqrt{2}), \tag{22}$$

where the Fermi energy $\varepsilon_F = p_F^2/2M^*_L$, and dimensionless coupling constant $\beta = \lambda_0 M^*_L/2\pi$. Taking the usual values of the dimensionless coupling constant $\beta \simeq 0.3$, and $(p_f - p_F)/p_F \simeq 0.2$, we get from Eq. (22) the large value of $\Delta_1(0) \sim 0.1\varepsilon_F$, while for normal metals one has $\Delta_1(0) \sim 10^{-3}\varepsilon_F$. Taking into account the omitted integral, we obtain

$$\Delta_1(0) \simeq 2\beta\varepsilon_F \frac{p_f - p_F}{p_F} \ln(1 + \sqrt{2}) \left(1 + \beta \ln \frac{2\omega_D}{E_0}\right). \tag{23}$$

It is seen from Eq. (23) that the correction due to the second integral is small, provided $E_0 \simeq 2\omega_D$. Below we show that $2T_c \simeq \Delta_1(0)$, which leads to the conclusion that there is no isotope effect since Δ_1 is independent of ω_D. But this effect is restored as $E_0 \to 0$. Assuming $E_0 \sim \omega_D$ and $E_0 > \omega_D$, we see that Eq. (21) has no standard solutions $\Delta(p) = \Delta_1(T = 0)$ because $\omega_D < \varepsilon(p \simeq p_f) - \mu$ and the interaction vanishes at these momenta. The only way to obtain solutions is to restore the condition $E_0 < \omega_D$. For instance, we can define the momentum $p_D < p_f$ such that

$$\Delta_1(0) = 2\beta\varepsilon_F \frac{p_D - p_F}{p_F} \ln(1 + \sqrt{2}) = \omega_D, \tag{24}$$

while the other part in the L_{FC} range can be occupied by a gap Δ_2 of the different sign, $\Delta_1(0)/\Delta_2 < 0$. It follows from Eq. (24) that the isotope effect is presented, while the both gaps can have s-wave symmetry. A more detailed analysis will be published elsewhere.

At $T \simeq T_c$, Eqs. (19) and (20) are replaced by the equation, which is valid also at $T_c \leq T \ll T_f$ in accord with Eq. (9) [5]

$$M^*_{FC} \simeq p_F \frac{p_f - p_i}{4T_c}, \;\; E_0 \simeq 4T_c; \;\; \text{if} \;\; T_c \leq T: \;\; M^*_{FC} \simeq p_F \frac{p_f - p_i}{4T}, \;\; E_0 \simeq 4T. \tag{25}$$

Equation (21) is replaced by its conventional finite temperature generalization

$$1 = N_{FC}\lambda_0 \int_0^{E_0/2} \frac{d\xi}{\sqrt{\xi^2 + \Delta_1^2(T)}} \tanh \frac{\sqrt{\xi^2 + \Delta_1^2(T)}}{2T} + N_L\lambda_0 \int_{E_0/2}^{\omega_D} \frac{d\xi}{\sqrt{\xi^2 + \Delta_1^2(T)}} \tanh \frac{\sqrt{\xi^2 + \Delta_1^2(T)}}{2T}. \tag{26}$$

Putting $\Delta_1(T \to T_c) \to 0$, we obtain from Eq. (26)

$$2T_c \simeq \Delta_1(0), \tag{27}$$

with $\Delta_1(T = 0)$ being given by Eq. (21). By comparing Eqs. (19), (25) and (27), we see that M^*_{FC} and E_0 are almost temperature independent at $T \leq T_c$. In the same way, as it was done in Sec. II, we can conclude that E_0 does not change along the Fermi surface, while M^*_{FC} increases when moving from the nodal direction to the point $\bar{M}$. Now a few remarks are in order. One can define T_c as the temperature when $\Delta_1(T_c) \equiv 0$. At $T \geq T_c$, Eq. (26) has only the trivial solution $\Delta_1 \equiv 0$. On the other hand, T_c can be defined as a temperature at which the superconductivity vanishes. Thus, we deal with two different definitions, which can lead to two different temperatures T_c and T^* in case of the d-wave symmetry of the gap. It was shown [12,39] that in the case of the d-wave superconductivity, taking place in the presence of the FC, there exist a nontrivial solutions of Eq. (26) at $T_c \leq T \leq T^*$ corresponding to the pseudogap state. It happens when the gap occupies only such a part of the Fermi surface, which shrinks as the temperature increases. Here T^* defines the temperature at which $\Delta_1(T^*) \equiv 0$ and the pseudogap state vanishes. The superconductivity is destroyed at T_c, and the ratio $2\Delta_1/T_c$ can vary in a wide range and strongly depends upon the material's properties, as it follows from consideration given in [12,19,39]. Therefore, provided there exists the pseudogap above T_c, then T_c is to be replaced by T^*, and Eq. (27) takes the form

$$2T^* \simeq \Delta_1(0). \tag{28}$$

The ratio $2\Delta_1/T_c$ can reach very high values. For instance, in the case of $Bi_2Sr_2CaCu_2Q_{6+\delta}$, where the superconductivity and the pseudogap are considered to be of the common origin, $2\Delta_1/T_c$ is about

28, while the ratio $2\Delta_1/T^* \simeq 4$, which is also valid for various cuprates [40]. Thus, Eq. (28) gives good description of the experimental data. We remark that Eq. (21) gives also good description of the maximum gap Δ_1 in the case of the d-wave superconductivity, because the different regions with the maximum absolute value of Δ_1 and the maximal density of states can be considered as disconnected [41]. Therefore, the gap in this region is formed by attractive phonon interaction which is approximately independent of the momenta.

Consider now two possible types of the superconducting gap $\Delta(\mathbf{p})$ given by Eq. (17) and defined by interaction $\lambda_0 V(\mathbf{p}, \mathbf{p}_1)$. If this interaction is dominated by a phonon-mediated attraction, the even solution of Eq. (17) with the s-wave, or the $s + d$ mixed waves, will have the lowest energy. Provided the pairing interaction $\lambda_0 V(\mathbf{p}_1, \mathbf{p}_2)$ is the combination of both the attractive interaction and sufficiently strong repulsive interaction, the d-wave odd superconductivity can take place, see e.g. [41]. But both the s-wave even symmetry and d-wave odd one lead to the approximately same value of the gap Δ_1 in Eq. (21) [18]. Therefore, the non-universal pairing symmetries in high-T_c superconductivity is likely the result of the pairing interaction, and the d-wave pairing symmetry cannot be considered as essential to high-T_c in keeping with experimental findings [28–32]. In case, if there were only the d-wave pairing, the crossover from superconducting gap to pseudogap can take place, so that the superconductivity is destroyed at the temperature T_c, with the superconducting gap being smoothly transformed into the pseudogap which closes at some temperature $T^* > T_c$ [19,39]. In the case of the s-wave pairing we can expect the absence of the pseudogap phenomena in accordance with the experimental observation, see [32] and references therein.

We turn now to a consideration of the maximum value of the superconducting gap Δ_1 as a function of the density x of mobile charge carriers. Being rewritten in terms of x and x_{FC} related to the variables p_i and p_f by Eq. (12), Eq. (22) takes the form

$$\Delta_1 \propto \beta(x_{FC} - x)x. \tag{29}$$

Here we take into account that the Fermi level $\varepsilon_F \propto p_F^2$, the density $x \propto p_F^2$, and thus, $\varepsilon_F \propto x$. Considering the field induced superconductivity, we can safely assume that $T_c \propto \Delta_1$ because this technique allows the study of properties of metal in question as a function of x without inducing additional defects or disorder which can have a dramatic impact on the transition temperature. Then, instead of Eq. (29) we have

$$T_c^{\alpha\gamma} \propto \beta^\alpha \beta^\gamma (x_{FC} - x)x. \tag{30}$$

In Eq. (30), we made the natural change $\beta = \beta^\alpha \beta^\gamma$ since the coupling constant β is fixed by the properties of metal in question. Following reference [42], we take that hole doped metals differ from electron doped ones only in the magnitude of the coupling constant β^γ which is smaller in case of electron doped metals. Now it is seen that Eq. (30) coincides with Eq. (13) producing the universal optimal doping level $x_{opt} = x_{FC}/2 = x_1/2$ in line with the experimental facts. In our model, we have $x_{opt}/x_{FC} = (r_s^{opt}/r_{FC})^2 = 2$, taking the value $r_s^{opt} \simeq 10$, we obtain $r_{FC} \simeq 7.0$. This result is in a reasonable agreement with the experimental value $r_{FC} \sim 7.5$ corresponding to sharp increase of the effective mass [15]. In line with facts [34], it follows from Eq. (30) that among the hole doped fullerides, the T_c ratios for $C_{60}/CHBr_3$-$C_{60}/CHCl_3$-C_{60} have to be the same as in the case of the respective electron doped fullerides because the factor β^γ drops out of the ratios.

As an example of the implementation of the previous analysis let us consider the main features of a room-temperature superconductor. The superconductor has to be a quasi two-dimensional structure, presented by infinite-layer compounds or by field-induced superconductivity in gated structures. As

it follows from Eq. (22), $\Delta_1 \sim \beta\varepsilon_F \propto \beta/r_s^2$. Noting that FCQPT takes place in 3D systems at $r_s \sim 20$ and in 2D systems at $r_s \sim 8$ [7], we can expect that Δ_1 of 3D system comprises 10% of the corresponding maximum value of 2D superconducting gap, reaching values as high as 60 meV for underdoped crystals with $T_c = 70$ [43]. On the other hand, it is seen form Eq. (22), that Δ_1 can be even large, $\Delta_1 \sim 75$ meV, and one can expect $T_c \sim 300$ K in the case of the s wave pairing as it follows from the simple relation $2T_c \simeq \Delta_1$. In fact, we can safely take $\varepsilon_F \sim 300$ meV, $\beta \sim 0.5$ and $(p_f - p_i)/p_F \sim 0.5$. Thus, we can conclude that a possible room-temperature superconductor has to be the s-wave superconductor in order to get rid of the pseudogap phenomena, which tremendously reduces the transition temperature. The density x of the mobile charge carriers must satisfy the condition $x \leq x_{FC}$ and be flexible to reach the optimal doping level. It is worth noting that the coupling constant β has to be sufficiently big because FC giving rise to the order parameter $\kappa(\mathbf{p})$ does not produce the gap Δ by itself. For instance, the coupling constant can be enhanced by an intercalation as it is done for fullerides [34,42].

Now we turn to the calculations of the gap and the specific heat at the temperatures $T \to T_c$. It is worth noting that this consideration is valid provided $T^* = T_c$, otherwise the considered below discontinuity is smoothed out over the temperature range $T^* \div T_c$. For the sake of simplicity, we calculate the main contribution to the gap and the specific heat coming from the FC. The function $\Delta_1(T \to T_c)$ is found from Eq. (26) upon expanding the right hand side of the first integral in powers of Δ_1 and omitting the contribution from the second integral on the right hand side of Eq. (26). This procedure leads to the following equation [18]

$$\Delta_1(T) \simeq 3.4 T_c \sqrt{1 - \frac{T}{T_c}}. \tag{31}$$

Thus, the gap in the spectrum of the single-particle excitations has quite usual behavior. To calculate the specific heat, the conventional expression for the entropy S [9] can be used

$$S = 2 \int \left[f(\mathbf{p}) \ln f(\mathbf{p}) + (1 - f(\mathbf{p})) \ln(1 - f(\mathbf{p})) \right] \frac{d\mathbf{p}}{(2\pi)^2}, \tag{32}$$

where

$$f(\mathbf{p}) = \frac{1}{1 + \exp[E(\mathbf{p})/T]}; \;\; E(\mathbf{p}) = \sqrt{(\varepsilon(\mathbf{p}) - \mu)^2 + \Delta_1^2(T)}. \tag{33}$$

The specific heat C is determined by

$$\begin{aligned} C = T\frac{dS}{dT} \simeq 4\frac{N_{FC}}{T^2} \int_0^{E_0} f(E)(1 - f(E)) \left[E^2 + T\Delta_1(T)\frac{d\Delta_1(T)}{dT} \right] d\xi \\ + 4\frac{N_L}{T^2} \int_{E_0}^{\omega_D} f(E)(1 - f(E)) \left[E^2 + T\Delta_1(T)\frac{d\Delta_1(T)}{dT} \right] d\xi. \end{aligned} \tag{34}$$

When deriving Eq. (34) we again use the variable ξ and the densities of states N_{FC}, N_L, just as before in connection to Eq. (21), and use the notation $E = \sqrt{\xi^2 + \Delta_1^2(T)}$. Equation (34) predicts the conventional discontinuity δC in the specific heat C at T_c because of the last term in the square brackets of Eq. (34). Upon using Eq. (31) to calculate this term and omitting the second integral on the right hand side of Eq. (34), we obtain

$$\delta C \simeq \frac{3}{2\pi}(p_f - p_i)p_F. \tag{35}$$

In contrast to the conventional result when the discontinuity is a linear function of T_c, δC is independent of the critical temperature T_c because the density of state varies inversely with T_c as it follows from Eq. (25). Note, that deriving Eq. (35) we take into account the main contribution coming from the FC. This contribution vanishes as soon as $E_0 \to 0$ and the second integral of Eq. (34) gives the conventional result.

IV. THE LINESHAPE OF THE SINGLE-PARTICLE SPECTRA

Consider the lineshape $L(q,\omega)$ of the single-particle spectrum which is a function of two variables. Measurements carried out at a fixed binding energy $\omega = \omega_0$, where ω_0 is the energy of a single-particle excitation, determine the lineshape $L(q, \omega = \omega_0)$ as a function of the momentum q. We have shown above that M^*_{FC} is finite and constant at $T \leq T_c$. Therefore, at excitation energies $\omega \leq E_0$ the system behaves like an ordinary superconducting Fermi liquid with the effective mass given by Eq. (19) [5,12]. At $T_c \leq T$ the low energy effective mass M^*_{FC} is finite and is given by Eq. (9). Once again, at the energies $\omega < E_0$, the system behaves as a Fermi liquid, the single-particle spectrum is well defined, while the width of single-particle excitations is of the order of T [5,8]. This behavior was observed in experiments on measuring the lineshape at a fixed energy [23]. It is pertinent to note that recent measurements of the lineshape suggest that quasiparticle excitation even in the $(\pi, 0)$ region of the Brillouin zone of $Bi_2Sr_2CaCu_2Q_{8+\delta}$ (Bi2212) are much better defined then previously believed from earlier Bi2212 data [44]. We remark that our model is in accordance with these measurements suggesting that well-defined quasiparticles exist at the Fermi level.

The lineshape can also be determined as a function $L(q = q_0, \omega)$ at a fixed $q = q_0$. At small ω, the lineshape resembles the one considered above, and $L(q = q_0, \omega)$ has a characteristic maximum and width. At energies $\omega \geq E_0$, quasiparticles with the mass M^*_L come into play, leading to a growth of the function $L(q = q_0, \omega)$. As a result, the function $L(q = q_0, \omega)$ possesses the known peak-dip-hump structure [45] directly defined by the existence of the two effective masses M^*_{FC} and M^*_L [5,12]. To have more quantitative and analytical insight into the problem we use the Kramers-Krönig transformation to construct the imaginary part $\mathrm{Im}\Sigma(\mathbf{p}, \varepsilon)$ of the self-energy $\Sigma(\mathbf{p}, \varepsilon)$ starting with the real one $\mathrm{Re}\Sigma(\mathbf{p}, \varepsilon)$ which defines the effective mass [46]

$$\frac{1}{M^*} = \left(\frac{1}{M} + \frac{1}{p_F}\frac{\partial \mathrm{Re}\Sigma}{\partial p}\right) \Big/ \left(1 - \frac{\partial \mathrm{Re}\Sigma}{\partial \varepsilon}\right). \tag{36}$$

Here M is the bare mass, while the relevant momenta p and energies ε are subjected to the conditions: $|p - p_F|/p_F \ll 1$, and $\varepsilon/\varepsilon_F \ll 1$. We take $\mathrm{Re}\Sigma(\mathbf{p}, \varepsilon)$ in the simplest form which accounts for the change of the effective mass at the energy scale E_0:

$$\mathrm{Re}\Sigma(\mathbf{p}, \varepsilon) = -\varepsilon\frac{M^*_{FC}}{M} + \left(\varepsilon - \frac{E_0}{2}\right)\frac{M^*_{FC} - M^*_L}{M}\left[\theta(\varepsilon - E_0/2) + \theta(-\varepsilon - E_0/2)\right]. \tag{37}$$

Here $\theta(\varepsilon)$ is the step function. Note that in order to ensure a smooth transition from the single-particle spectrum characterized by M^*_{FC} to the spectrum defined by M^*_L the step function is to be substituted by some smooth function. Upon inserting Eq. (37) into Eq. (36) we can check that inside the interval $(-E_0/2, E_0/2)$ the effective mass $M^* \simeq M^*_{FC}$, and outside the interval $M^* \simeq M^*_L$. By applying the Kramers-Krönig transformation to $\mathrm{Re}\Sigma(\mathbf{p}, \varepsilon)$, we obtain the imaginary part of the self-energy [18]

$$\mathrm{Im}\Sigma(\mathbf{p},\varepsilon) \sim \varepsilon^2 \frac{M^*_{FC}}{\varepsilon_F M} + \frac{M^*_{FC} - M^*_L}{M}\left(\varepsilon \ln\left|\frac{\varepsilon + E_0/2}{\varepsilon - E_0/2}\right| + \frac{E_0}{2}\ln\left|\frac{\varepsilon^2 - E_0^2/4}{E_0^2/4}\right|\right). \quad (38)$$

We can see from Eq. (38) that at $\varepsilon/E_0 \ll 1$ the imaginary part is proportional to ε^2; at $2\varepsilon/E_0 \simeq 1$ $\mathrm{Im}\Sigma \sim \varepsilon$; at $E_0/\varepsilon \ll 1$ the main contribution to the imaginary part is approximately constant. This is the behavior that gives rise to the known peak-dip-hump structure. Then, it is seen from Eq. (38) that when $E_0 \to 0$ the second term on the right hand side tends to zero, the single-particle excitations become better defined resembling that of a normal Fermi liquid, and the peak-dip-hump structure eventually vanishes. On the other hand, the quasiparticle amplitude $a(\mathbf{p})$ is given by [46]

$$\frac{1}{a(\mathbf{p})} = 1 - \frac{\partial \mathrm{Re}\Sigma(\mathbf{p},\varepsilon)}{\partial\varepsilon}. \quad (39)$$

It follows from Eq. (36) that the quasiparticle amplitude $a(\mathbf{p})$ rises as the effective mass M^*_{FC} decreases. Since, as it follows from Eq. (12), $M^*_{FC} \sim (p_f - p_i)/p_F \sim (x_{FC} - x)/x_{FC}$, we are led to a conclusion that the amplitude $a(\mathbf{p})$ rises as the doping level rises, and the single-particle excitations become better defined in highly overdoped samples. It is worth noting that such a behavior was observed experimentally in so highly overdoped Bi2212 that the gap size is about 10 meV [17]. Such a small size of the gap testifies that the region occupied by the FC is small since $E_0/2 \simeq \Delta_1$. Quasiparticles located at the intersection of the nodal direction of the Brillouin zone with the Fermi level should also be well-defined comparatively to quasiparticles located at the point $\bar{M}$ because of the decrease of the effective mass M^*_{FC} when moving from the point $\bar{M}$ to the nodal direction, as it was discussed in Sections II and III. This is especially true in regard to strongly underdoped samples.

V. SUMMARY

In conclusion, we have shown that the theory of high temperature superconductivity based on the fermion-condensation quantum phase transition and on the conventional theory of superconductivity permits to describe high values of T_c, T^* and of the maximum value of the gap Δ_1, which may be as big as $\Delta_1 \sim 0.1\varepsilon_F$ or even larger. We have also traced the transition from conventional superconductors to high-T_c and demonstrated that in the highly overdoped cuprates the single-particle excitations become much better defined, resembling that of a normal Fermi liquid.

We have also shown by a simple, self-consistent analysis, that the general features of the shape of the critical temperature T_c as a function of the density x of the mobile carriers in the metals and the value of the optimal doping x_{opt} can be understood within the framework of the theory of the high-T_c superconductivity based on FCQPT. We have demonstrated that neither the d-wave pairing symmetry, nor the pseudogap phenomenon, nor the presence of the Cu-O planes are of importance for the existence of the high-temperature superconductivity. As a result, we can conclude that the generic properties of high-temperature superconductors are defined by the 2D charge (electron or hole) strongly correlated liquid rather then by solids which hold this liquid. While the solids arrange the presence of the pseudogap phenomena, the s-wave pairing symmetry or d-wave one, the electron-phonon coupling constant defining T_c, the variation region of x, and so on. The main features of a room-temperature superconductor have also been outlined.

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